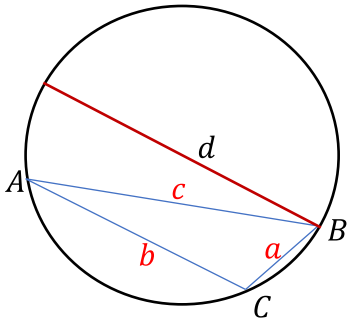
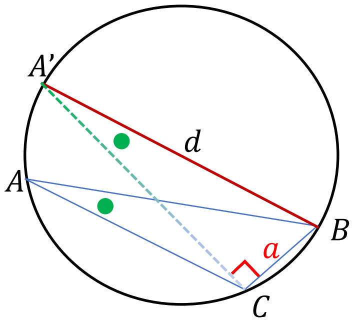
The Sine Rule (also known as the *Law of Sines*) as applied to the triangle ABC with opposite sides a, b and c is typically presented in mathematics classes as either the equality given as $\frac{a}{\sin A} = \frac{b}{\sin B}$ or equivalently $\frac{\sin A}{a} = \frac{\sin B}{b}$ or even more fully as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
I suspect that it's rarely expressed as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = d$, and yet this last expression is arguably more pedagogically meaningful because it explicitly speaks to Euclid's first common notion that *things that are equal to the same thing are equal to each other*.

In the Sine Rule this 'thing', d, actually has a special significance. Remarkably, it is the diameter of the triangle's circumscribing circle as shown here.



We can demonstrate that the diameter has the length $\frac{a}{\sin A}$ by drawing in a second triangle $A'BC$ as shown here.



The angles at A and A' are equal since they are both subtended by the chord BC. Because triangle $A'BC$ is right angled (Thales theorem) we immediately have that $\sin A' = \frac{a}{d}$ and thus $d = \frac{a}{\sin A'} = \frac{a}{\sin A}$. In the same way, other diameters can be drawn to show the cases for the other two angles in the triangle - namely that $d = \frac{b}{\sin B} = \frac{c}{\sin C}$.

There is an infinite number of inscribed triangles that can be drawn within any circle of diameter d. Irrespective of the size of their internal angles, they all exhibit the same property that the ratio of the three sides to the sines of the opposite angles is the constant d. Moreover, this circle uniquely defines the set of all triangles that express this particular ratio.

The circle then is a sort of 'locus of equality' and the size of it determines the size of d.

Challenge 11: If $d = 12$ and the side b is an integer different to d, then given the angle B is a whole number of degrees, what value must b be?