In some countries a game of chance called Lotto is played commercially. In one form of the game six balls are drawn randomly (without replacement) from a barrel containing $45$ numbered balls. The main prize is won when a player's ***Select-6*** entry matches the drawn numbers. Alternatively, the player can buy a ***Systems-n*** entry allowing the selection of up to $n = 15$ numbers, with each extra number beyond six costing more. Any six of the $n$ numbers could be the winning numbers.

Although on taking a *Systems*-$15$ entry a player may well accept that the chance of winning the main prize is far less than $\frac{15}{45} = \frac{1}{3}$, they might nevertheless reason that there is about a one-third chance of each drawn number being one of theirs, and that consequently they would only have to be lucky six times in a row to win the main prize.

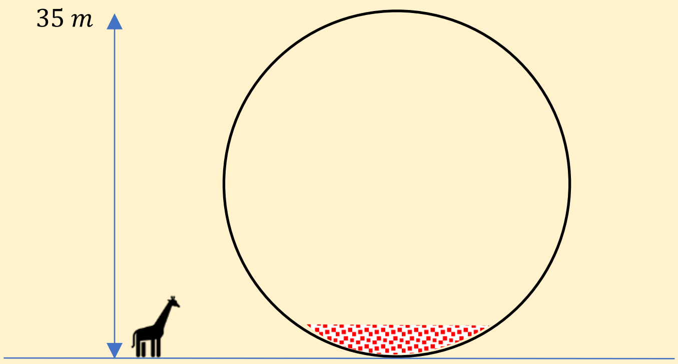
'Lucking in' like that doesn't seem too improbable considering the possible payout that might occur. It is an understandable perception for a player to have. The illusion of reasonableness is intensified by the fact that the $45$ numbered balls occupy such a small space inside the glass barrel. The game is designed to look easy.

In fact, of the $8{,}145{,}060$ possible sets of six numbers that can be drawn from the barrel, a *Systems*-$15$ entry includes $5{,}005$ of them. Therefore, for a cost of over $5{,}000$ times the price of a *Select-6* entry, the probability of winning the main prize is $\frac{5005}{8145060}$, still only about one chance in $1627$. If it were possible, a *Systems-41* ticket would be required to have a slightly better than even money chance of winning the main prize.

Human beings are generally not good at comprehending the size of numerical expressions, particularly those that occur in probability calculations that involve factorials (even when they know how to calculate them). This lack of comprehension is at the heart of the grand illusion of the game of Lotto. In a *Select-6* entry, players see $45$ balls, and bet on $6$ of them. They don't perceive that it's their entry against the $8{,}145{,}059$ others.

The ball selection is randomised by jets of air blowing upwards through the lightweight balls and there is plenty of empty tumbling space enabling the balls to mix well in the barrel. If each ball measures $3$ cm in radius, then, with a typical barrel diameter of about $60$ cm, the $45$ balls together would occupy about $4.5\%$ of the available barrel space.

If lotto syndicates acted transparently, they might have a ball for each *Select-6* entry. (For example, a unique selection of $6$ numbers could be written around each ball's circumference). Then, with $8{,}145{,}060$ balls occupying $4.5\%$ of the space, the barrel would need to be about $35$ metres high.




*Challenge 19: Show that there's about a 13% chance of missing the main prize with a Systems 44.*