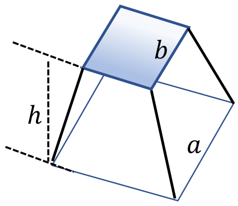
Does the formula I just established make sense?
Let's accept that the volume of a truncated square pyramid is $V = \frac{1}{3}h(a^2 + ab + b^2)$ where a and b are the lengths of the bottom and top edges, and h is the perpendicular height.



Some interesting aspects of this formula are its form and its symmetry. Exploring its form, we note that when $b = 0$, the resulting pyramid has a volume of $\frac{1}{3}a^2h$, and when $a = 0$ the now inverted pyramid has a volume of $\frac{1}{3}b^2h$. When $a = b$ we have a square prism with a volume of a^2h. The formula is symmetric in the sense that swapping a and b around makes no essential difference.

A great teaching strategy is to spend a little classroom time playing with such derived formulae, discussing why they makes sense and asking 'what-if' questions around them.

As an illustration, the Cosine rule states that, for any triangle ABC with opposite sides a, b and c, $c^2 = a^2 + b^2 - 2ab \cdot \cos C$. If the angle $C > 90°$ the side c facing it is larger than it would be had C been 90°. Thus, the formula reflects the fact that the cosine of obtuse angles are negative quantities. Pushing it further, if C is allowed to be 180°, the formula collapses to $c^2 = a^2 + b^2 + 2ab = (a + b)^2$ so that $c = a + b$ (as expected). If C went the other way, to 0°, then, in a similar way, c would become the positive difference between a and b. Of course, when C is 90°, Pythagoras' theorem appears.

As a third example, consider the symmetric formula $N_e = \frac{4mf}{m+f}$ where N_e is the *effective population* of a contained population of $N = m + f$ members of a species, where m and f represent the numbers of males and females. The word *effective* means to be successful in producing some desired result, but what can be made of it in this context? How does the formula relate to the idea of effectiveness?

Firstly, swapping the numbers of males and females will not change N_e. Further, since $N = m + f$ we can write $N_e = \frac{4mf}{N}$. The product mf is maximised when $m = f$, whence $N_e = \frac{4m^2}{2m} = 2m = N$. (This may explain the constant 4 in the formula.) Any imbalance of the sexes necessarily reduces N_e. For example, if $m = 2f$ then $N_e = \frac{4mf}{N} = \frac{8f^2}{3f} = \frac{8}{3}f < 3f$, but $\max(N_e) = N = 3f$. Observe that the number of possible distinct couplings of males and females is mf. So, the word *effective* may be to do with the generational potential of the species. Finally, by dividing by N we can write $\frac{N_e}{N} = 4 \times \left(\frac{m}{N}\right) \times \left(\frac{f}{N}\right)$ and this form provides a clearer picture of the concept and also suggests how it may have been designed.

Challenge 1: The slant height s of the truncated pyramid is given by $s = \sqrt{h^2 + 2\left(\frac{a-b}{2}\right)^2}$. Comment on the form and symmetry of this formula.