An old banker's rule called **the Rule of 72** went something like this. If a client invested an amount P at a compounding interest rate of r % p.a., then in $\frac{72}{r}$ years the return to them would be about $2P$. For example, an investment of \$200, yielding 6% p.a., has a future value of about \$400 in 12 years and we can make a quick check of this by applying the compound interest formula.
Thus $A = 200\left(1+\frac{6}{100}\right)^{12} \approx \$402$.

If we set $Q = \left(1+\frac{r}{100}\right)^{\frac{72}{r}}$ then, for various values of r, we can tabulate a few of its values to see how useful the rule might be.

r	3 %	6 %	8 %	9 %	12 %	24 %
Q	2.03	2.01	1.999	1.992	1.97	1.91

The online computer program *Wolfram Alpha™* was used to show that the solution for r in the equation $Q = \left(1+\frac{r}{100}\right)^{\frac{72}{r}} = 2$ (an exact doubling) is approximately $r = 7.847$.

The rule of 72 was widely in use prior to the availability of electronic computing devices. Its popularity may have been partly due to the fact that $72 = 2^3 \times 3^2$ has $(3+1)(2+1) = 12$ factors. In the formula, 72 is divided by the interest rate. So, because 72 is highly composite, practical estimates could be made quickly for a range of whole number interest rates.

A possible pedagogical investigation

We might note that the number of years it would take for an investment to quadruple would be twice that for the same investment to double. Therefore, by applying the rule of 72 twice, an amount A invested at r % p.a. would take approximately $2 \times \frac{72}{r} = \frac{144}{r}$ years to grow to $4A$.

More generally, it would take approximately $k \times \frac{72}{r}$ years to grow A to $2^k A$. Setting $2^k = m$, so that $k = \frac{\log m}{\log 2}$, it becomes clear that growing A to mA will take approximately $\frac{72 \log m}{r \cdot \log 2}$ years.

For example, a rule of thumb to estimate the time to triple an investment might be something like $\frac{72 \log 3}{r \cdot \log 2} \cong \frac{114}{r}$ but 114 has only 2 factors and so such a rule would be fairly impractical.

However, the author did find at least one that might be worthy of consideration. A ten-fold investment rule of thumb is determined as $\frac{72 \log 10}{\log 2} \cong \frac{239.18}{r} \cong \frac{240}{r}$.

The number 240 has 20 factors, many of which might represent typical modern-day interest rates, perhaps up to about 12% p.a. Using the new **Ten-fold Rule of 240**, an investment of A, at say 6 % p.a., would grow to about $10A$ in 40 years. The actual time required would be 39.52 years.

Challenge 16: Investigate how good the rule of 240 is for other interest rates in the suggested range.